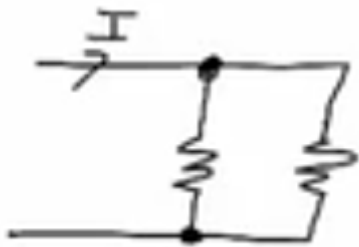


Parallel Circuits and Resistance

Recall That:

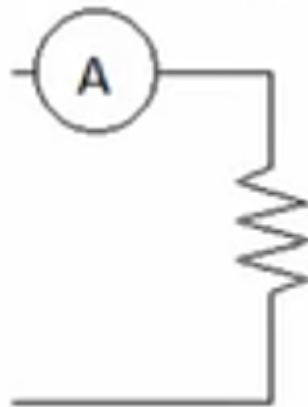
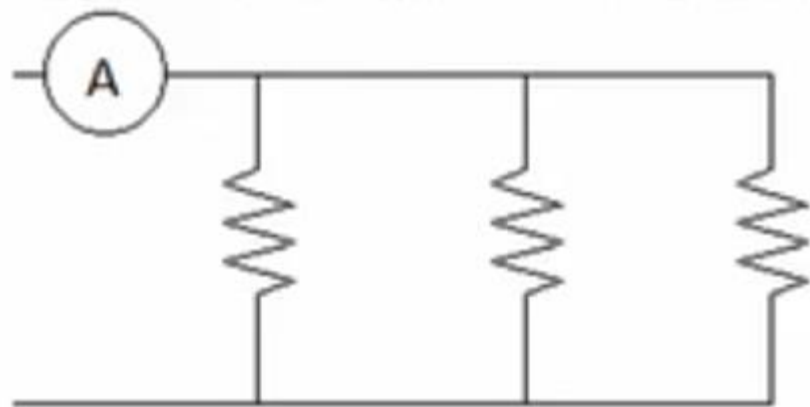
Elements in a circuit are in parallel if they are along different current paths but share common junctions

A parallel circuit is where all elements of the circuit are on their own branch, in parallel with each other



Resistors in parallel

When multiple resistors are placed in parallel, their individual resistances have a combined effect on the current



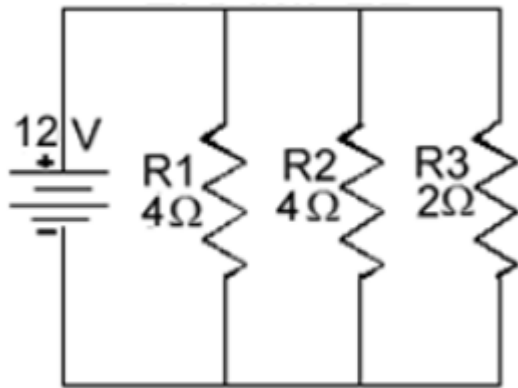
Multiple resistors in parallel can be combined into a single resistor with an equivalent resistance for the purposes of circuit analysis

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \dots$$

$$R_T = \left(\frac{1}{R_1} + \frac{1}{R_2} + \dots \right)^{-1}$$

Resistance Formula

- You can find TOTAL RESISTANCE in a Parallel circuit with the following formula: $1/R_t = 1/R_1 + 1/R_2 + 1/R_3 + \dots$



Voltage = 12V
R1 = 4 Ohm
R2 = 4 Ohm
R3 = 2 Ohm

$$\frac{1}{R_t} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$
$$\frac{1}{R_t} = \frac{1}{4} + \frac{1}{4} + \frac{1}{2}$$

- It is easiest to change the fractions into decimal numbers (eg 1 divide by 4 equals .25):
- $1/R_t = .25 + .25 + .5$
- $1/R_t = 1$
- Now you have to get rid of the 1 on the left side so...
- $R_t = 1/1$
- $R_t = 1 \text{ Ohms}$

Worked Example

- A parallel circuit with resistors of 30 Ohms, 60 Ohms, 20 Ohms and 10 Ohms. Then the total resistance is:

$$\begin{aligned} \frac{1}{R_t} &= \frac{1}{30} + \frac{1}{60} + \frac{1}{20} + \frac{1}{10} \\ &= \frac{2}{60} + \frac{1}{60} + \frac{3}{60} + \frac{6}{60} \\ &= \frac{12}{60} = \frac{1}{5} \\ R_t &= \frac{1}{1/5} = 5 \text{ Ohms} \end{aligned}$$

Generic Calculator input

30 [1/x] [+] 60 [1/x] [+] 20 [1/x] [+] 10 [1/x] [=] [1/x]

Product-over-sum rule

$$R_t = \frac{R_1 * R_2}{R_1 + R_2}$$

`<- their product`
`<- over`
`<- their sum`

Eg 30-Ohm resistor in parallel with a 60-Ohm resistor, the total resistance would be:

$$R_t = \frac{30 * 60}{30 + 60}$$

$$= \frac{1800}{90} = 20 \text{ Ohms}$$

More than 2 resistors. For 3 resistors, the rule becomes:

$$R_t = \frac{R_1 * R_2 * R_3}{R_1 * R_2 + R_1 * R_3 + R_2 * R_3}$$

A more complex one!

- Voltage = 12 Volts
- R1 = 10 Ohms
- R2 = 20 Ohms
- R3 = 10 Ohms
- R4 = 1 Ohms
- $1/R_t = 1/10 + 1/20 + 1/10 + 1/1$
- $1/R_t = .1 + .05 + .1 + 1$
- $1/R_t = 1.25$
- $R_t = 1/1.25 = .8 \text{ Ohms}$

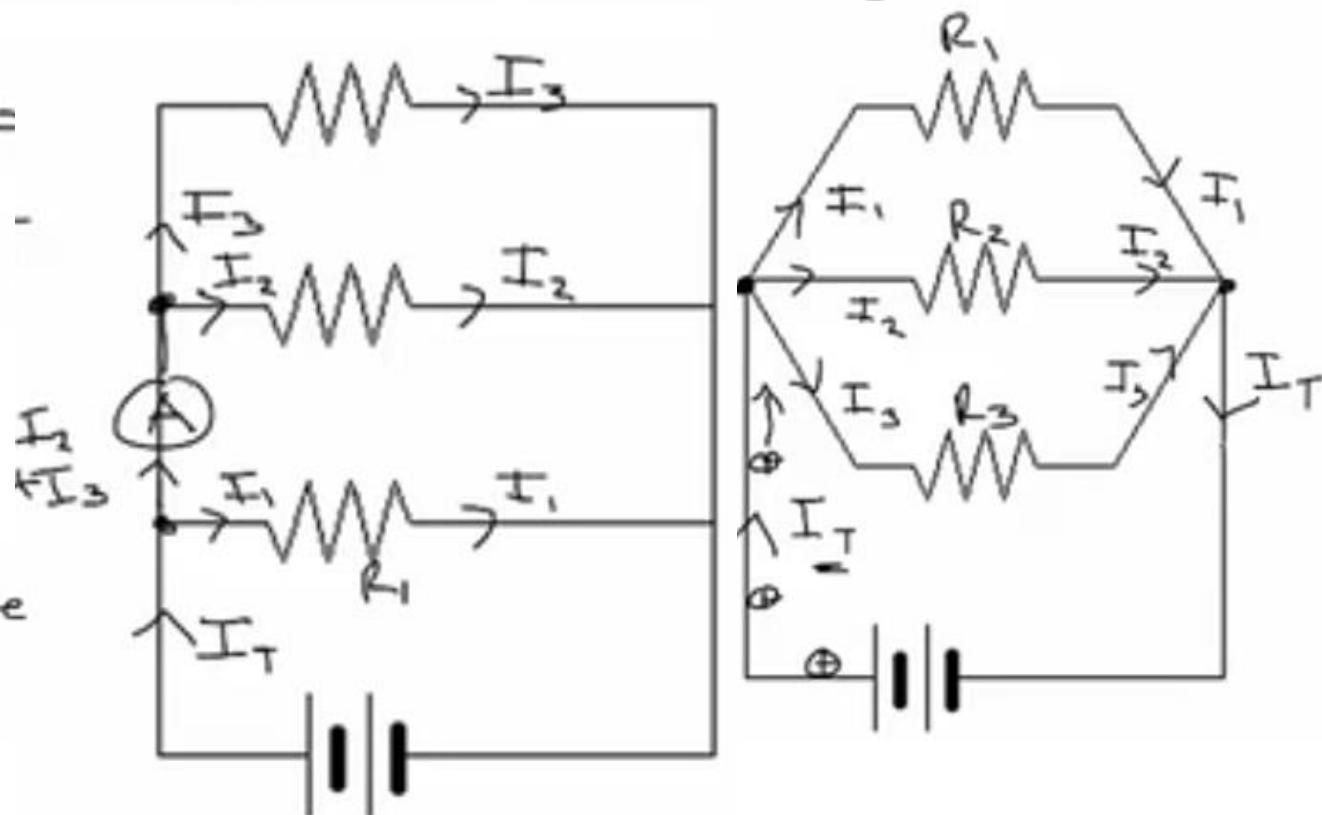
Kirchoff's current law

The sum of the currents flowing into a junction is equal to the sum of the currents flowing out

$$\sum I_{in} = \sum I_{out}$$

$$I_T = I_1 + I_2 + I_3$$

Current follows the path of least R.

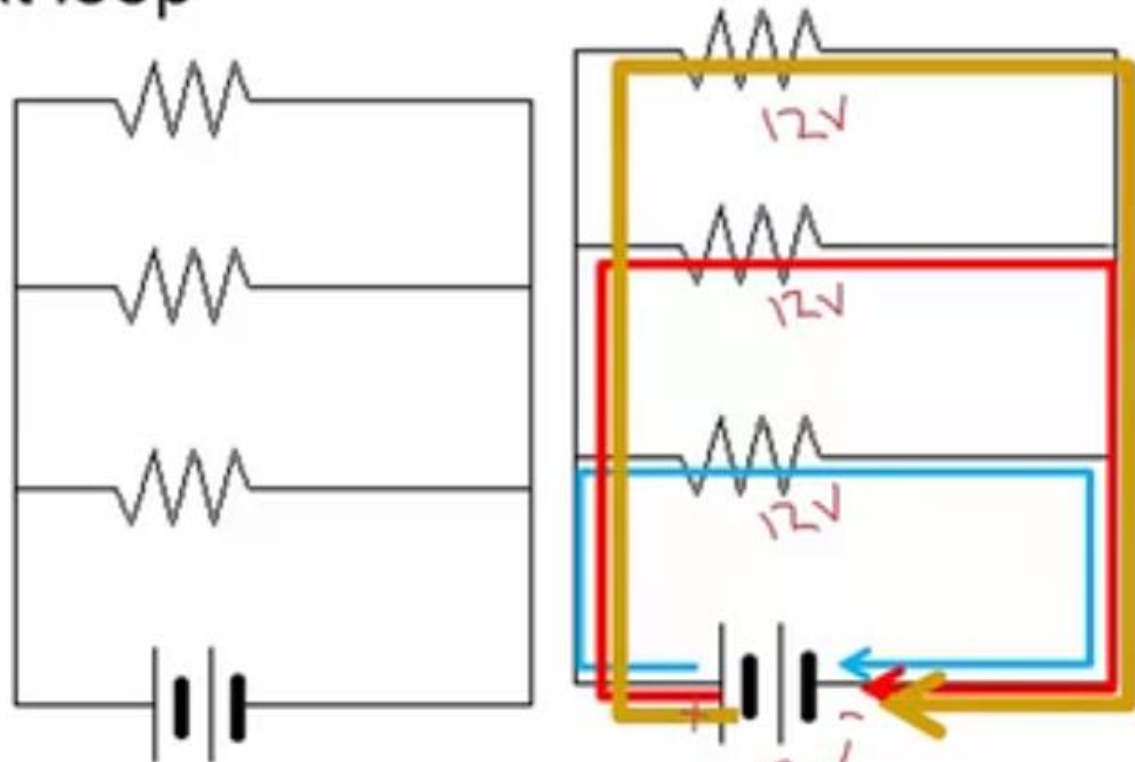


Kirchoff's voltage law

The potential drop around a loop in a circuit is equal to the emf supplied to that loop



$$(2v + 6v + 4v = 12v)$$



Each branch of a parallel circuit has the same potential difference

Circuit Analysis

Ohm's law

Equivalent resistance

Kirchoff's voltage law

Kirchoff's current law

Example problem

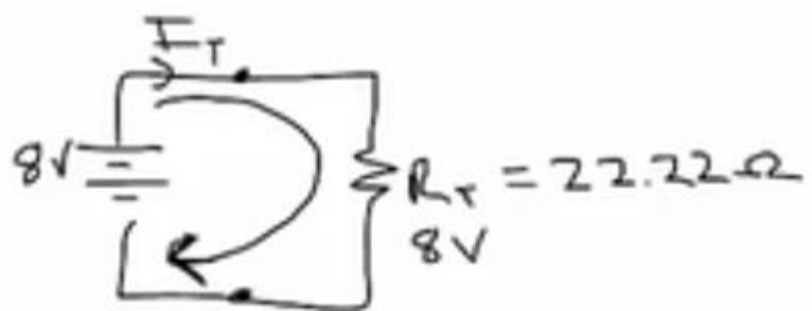
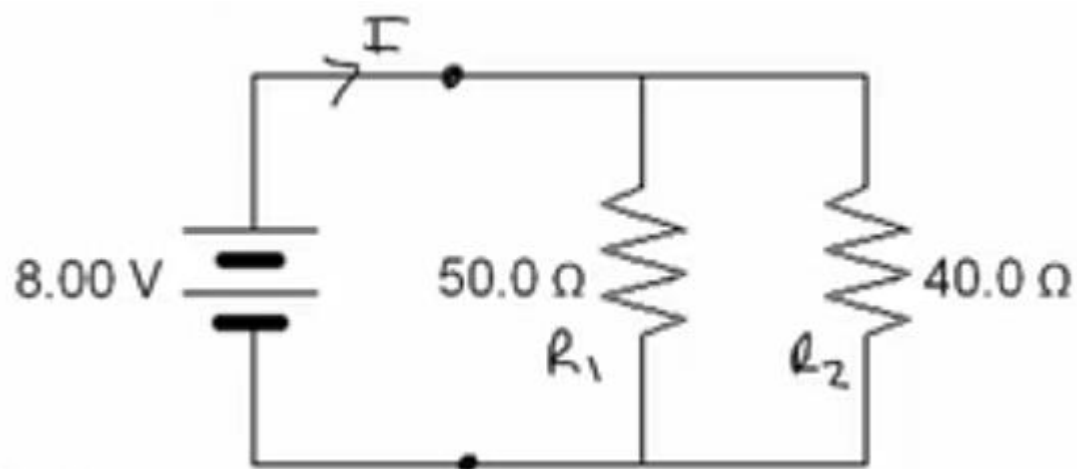
Find the current produced by the battery.

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$R_T = \left(\frac{1}{R_1} + \frac{1}{R_2} \right)^{-1}$$

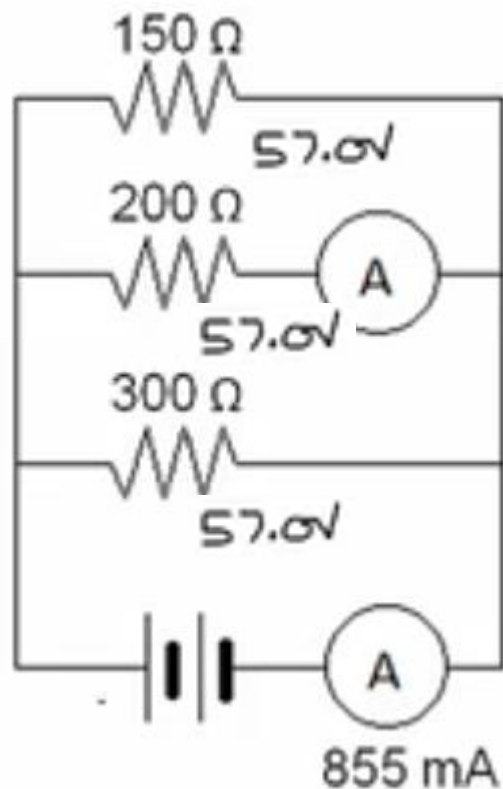
$$= \left(\frac{1}{50} + \frac{1}{40} \right)^{-1} = 22.22 \Omega$$

$$I_T = \frac{V_T}{R_T} = \frac{8}{22.22} = 0.360 \text{ A}$$

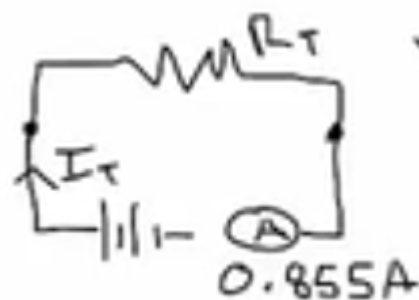


Worked Example 2

Find the current flowing through the middle branch of the parallel circuit.



$$R_T = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right)^{-1}$$
$$= \left(\frac{1}{150} + \frac{1}{200} + \frac{1}{300} \right)^{-1} = 66.67 \Omega$$

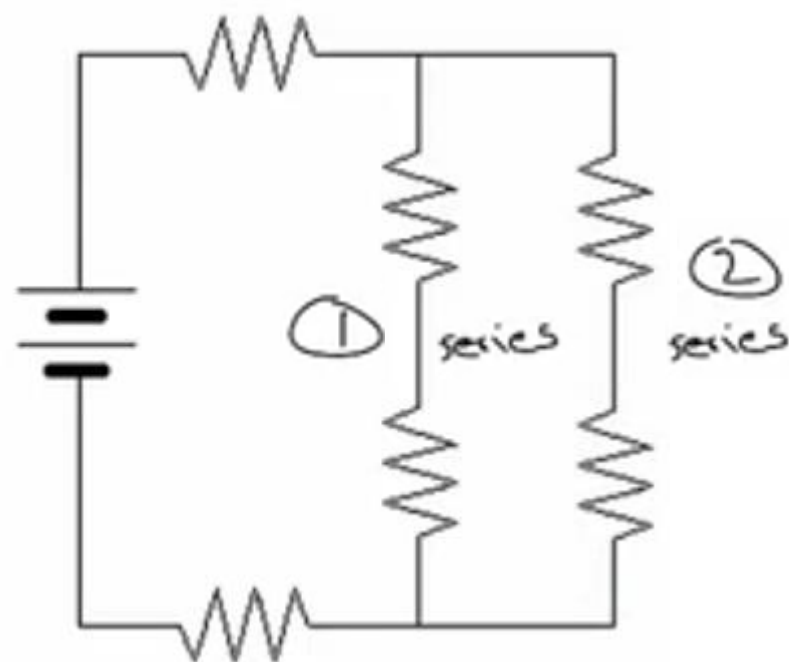


$$V_T = I_T R_T$$

$$= 0.855 \times 66.67 = 57.0 \text{ V}$$

Combination circuits

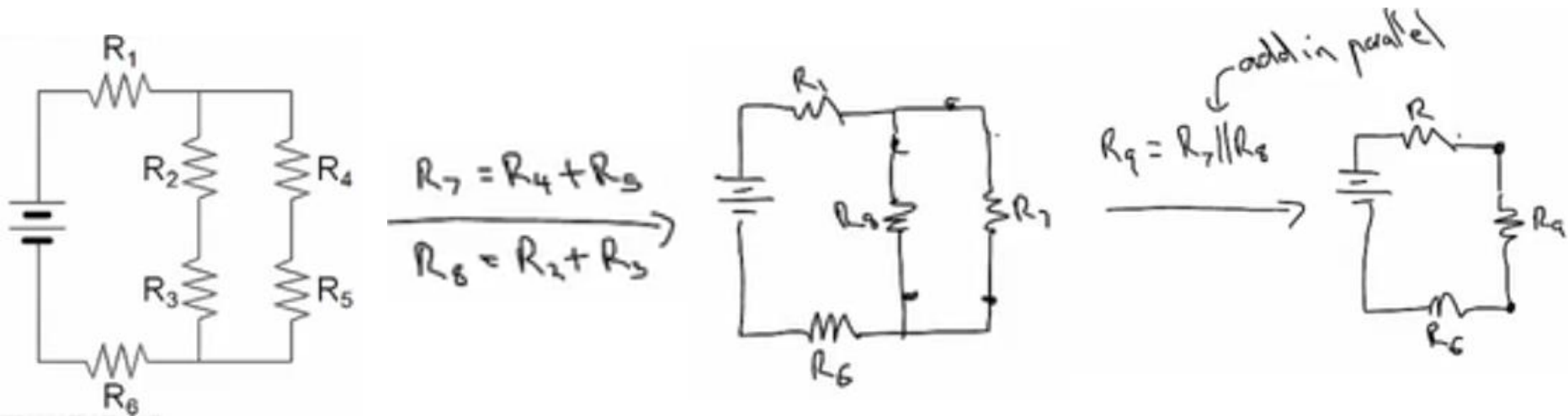
Combination circuits contain a combination of series elements and parallel elements



(1) & (2) in parallel

Equivalent resistance

To find the equivalent resistance of a combination circuit, reduce the series and parallel components of the circuit step by step until left with a single, equivalent resistance



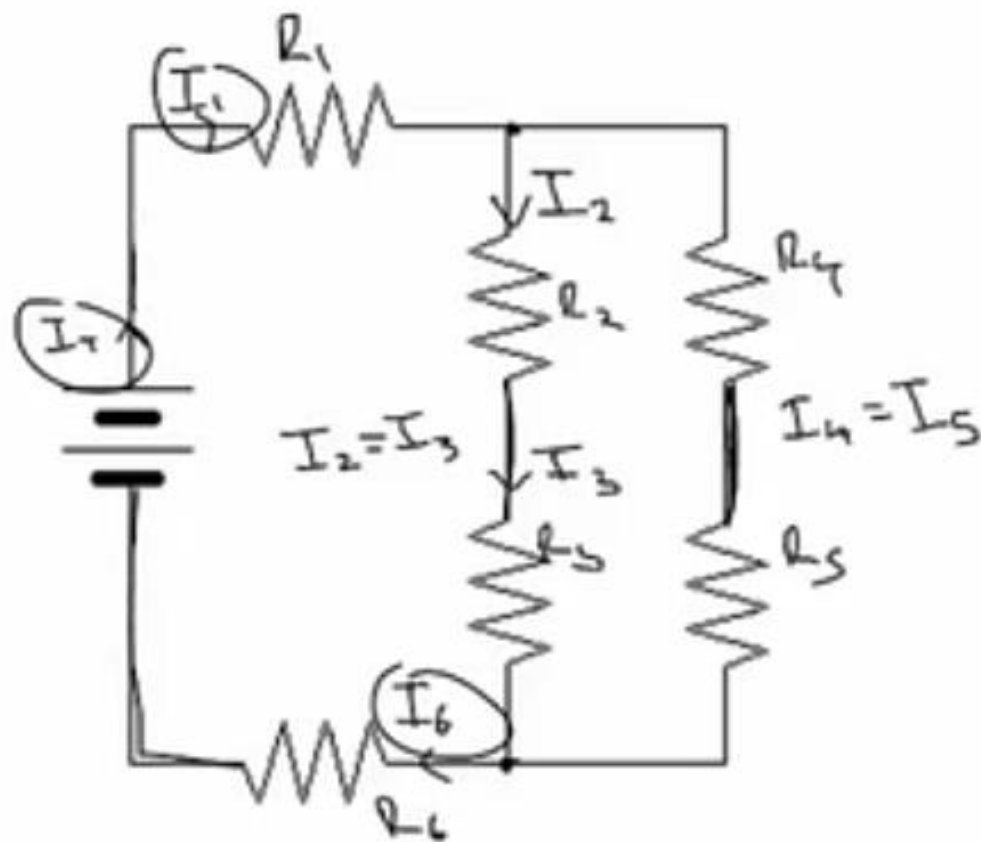
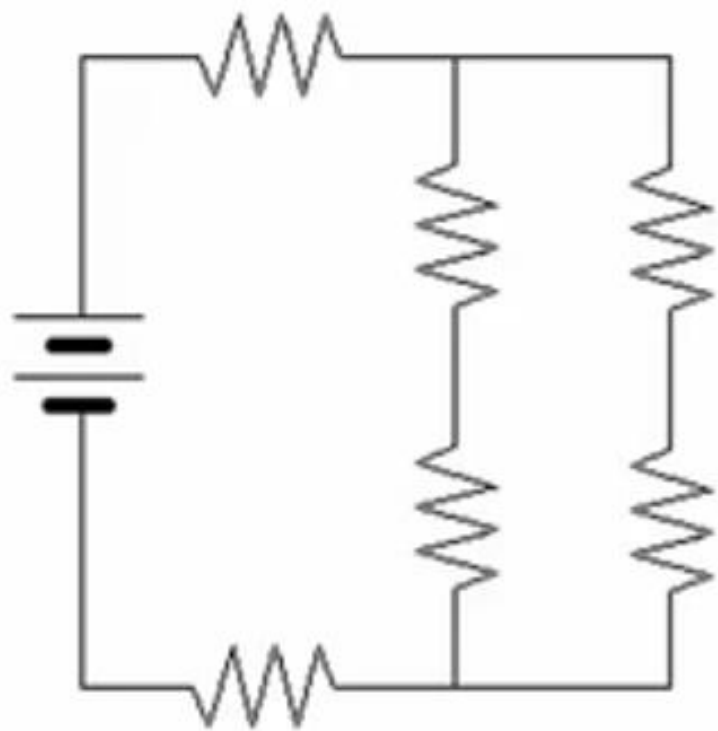
Redrawing parts of the circuit diagram will help to keep track of the step by step reduction of resistors



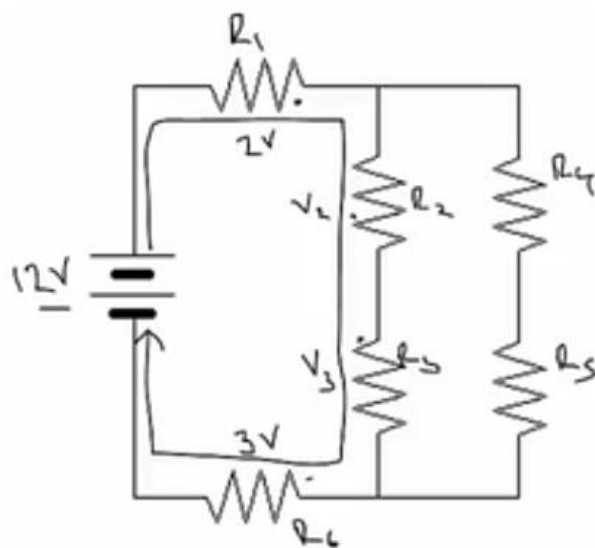
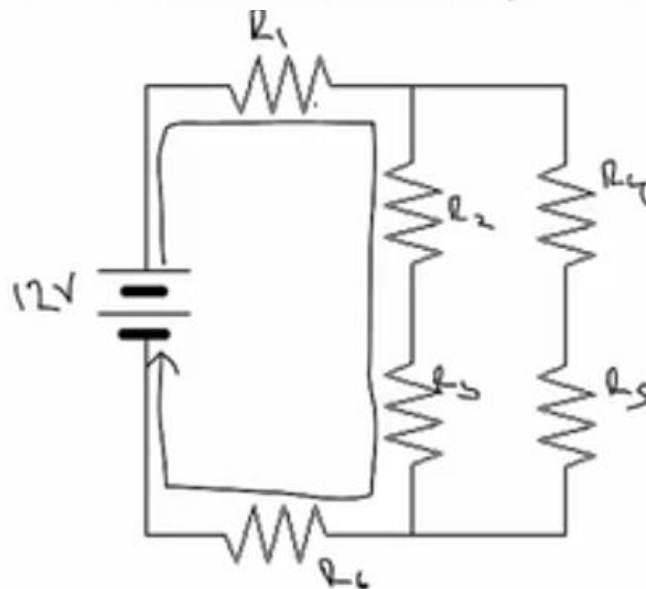
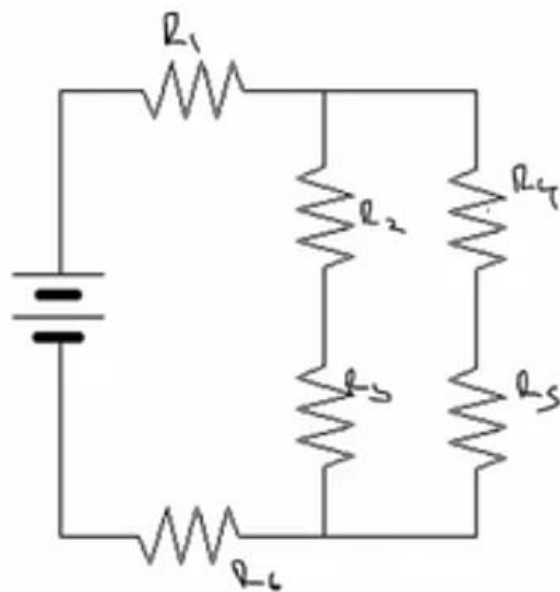
$$R_T = R_1 + R_9 + R_6$$

Kirchoff's laws in combination circuits

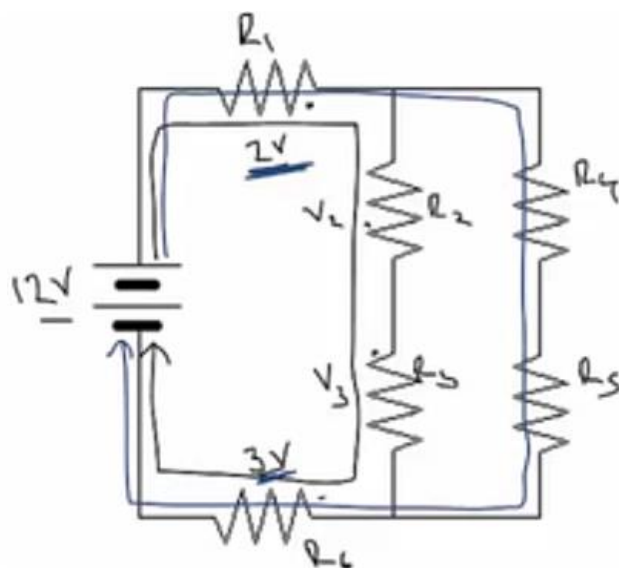
The current remains the same between **elements in series** with each other



The voltage drop of each **entire branch** of a parallel section is identical



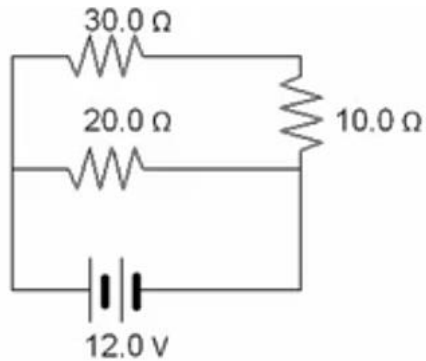
$$V_2 + V_3 = 7V$$



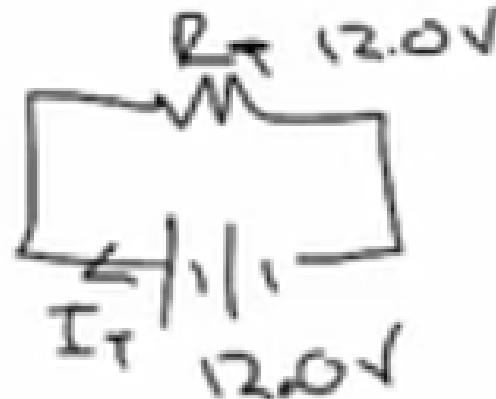
$$V_4 + V_5 = 7V$$

Worked Example

Find the current produced by the battery.



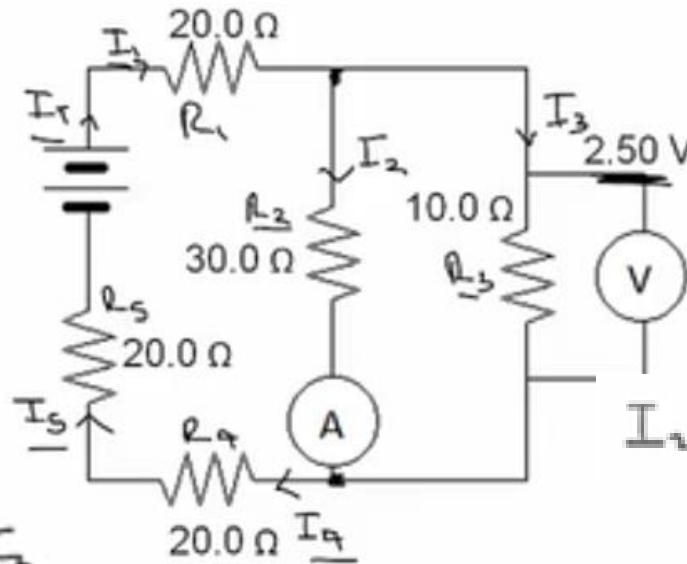
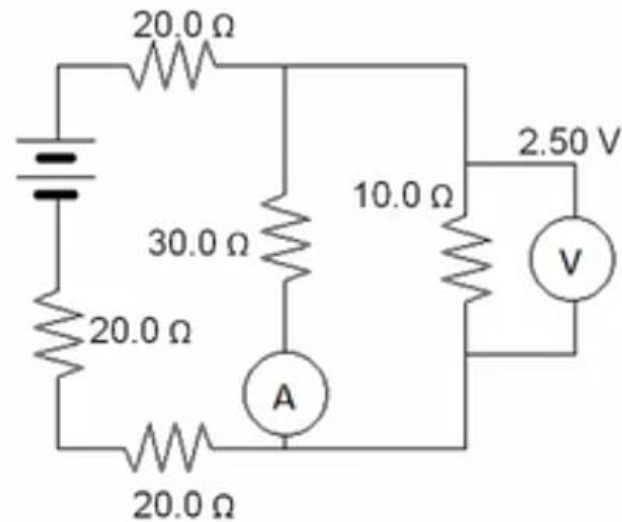
$$R_T = \left(\frac{1}{40} + \frac{1}{20} \right)^{-1}$$
$$= 13.33\Omega$$



$$I_T = \frac{V_T}{R_T} = \frac{12.0}{13.33} = \underline{\underline{0.900A}}$$

Worked Example

Find the emf supplied by the battery.



$$I_3 = \frac{V_3}{R_3} = \frac{2.50}{10.0} = 0.250 \text{ A}$$

$$I_2 = \frac{V_2}{R_2} \quad (V_2 = V_3)$$

$$= \frac{2.50}{30.0} = 0.0833 \text{ A}$$

$$I_T = I_2 + I_3$$

$$V_1 = I_1 R_1 = 0.2333 \times 20 = 6.667 \text{ V}$$

$$V_4 = V_5 = V_1 = 6.667 \text{ V}$$

$$V_T = V_1 + V_2 + V_4 + V_5 + V_3$$

$$= 6.667 + 2.50 + 6.667 + 6.667 + 2.50$$