Term 3 Week 3

Q1

In a small nuclear power plant one fifth of the fission energy is converted into a useful output power of 10 MW. If the average energy released per fission is 3.2×10^{-11} J, calculate the number of uranium-235 nuclei which will undergo fission per day.

one-fifth efficiency so total output $(= 10 \times \frac{100}{20} = 50(\text{MW})$ **(1)**

energy in one day $= 50\times 10^{6} \times 24 \times 3600(\text{J})$ **(1)** $(4.32 \times 10^{12}\ \text{J})$

fission atoms per day $= \frac{4.32\times10^{12}}{3.2\times10^{-11}} = 1.35 \times10^{23}$ **(1)**

Q2

$$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{92}_{36}Kr + ^{141}_{56}Ba + N \text{ neutrons}$$

Calculate N. $N = 3$

Calculate the energy released in MeV when one uranium nucleus undergoes fission in this reaction. Use the following data.

mass of neutron	=	1.00867 u
mass of ^{235}U nucleus	=	234.99333 u
mass of ^{92}Kr nucleus	=	91.90645 u
mass of ^{141}Ba nucleus	=	140.88354 u

1 u is equivalent to 931 MeV

$\Delta m = 234.99333 - (91.90645 + 140\ 88354) - (2 \times 1.00867)$ **(1)**

$= 0.186$ u **(1)**

(if last term in Δm omitted or incorrect number of neutrons used in calculation, treat answer as C.E.)

energy released $= 0.186 \times 931 = 173$ MeV **(1)**

Q3

A radioactive source has an activity of 3.2×10^9 Bq and emits α particles, each with kinetic energy of 5.2 Me V. The source is enclosed in a small aluminium container of mass 2.0×10^{-4} kg which absorbs the radiation completely.

energy absorbed per sec (= energy released per sec)

$= 3.2 \times 10^9 \times 5.2 \times 10^6 \times 1.6 \times 10^{-19}$ **(1)**

$= 2.7 \times 10^{-3}$ (J) **(1)** (2.66×10^{-3} (J))

- proton mass, $m_p = 1.673 \times 10^{-27}$ kg
- neutron mass, $m_n = 1.675 \times 10^{-27}$ kg
- mass of a nucleus = 6.643×10^{-27} kg
- The mass of a nucleus is **less** than the sum of the masses of its parts; this is true for **all** nuclides.
- So much for conservation of mass!
- Atomic mass unit (amu, or u) as a convenient unit of nuclear mass.
- 1 amu or 1 u = 1/12 the mass of a neutral ^{12}C atom (i.e. including its six electrons) = 1.66056×10^{-27} kg. Thus:
- $m_p = 1.0073$ u
- $m_n = 1.0087$ u
- $m_e = 0.00055$ u
- mass of a neutral atom = 4.0026 u

- Measuring an object’s mass, we are determining its energy.
- A helium nucleus has less mass than its constituent nucleons; in pulling them apart, we do work and so give them energy; hence their mass is greater.)

Q4

- H: 1.0073 atomic mass units
- · Li: 7.0160 atomic mass units
- · He: 4.0015 atomic mass units.
- An atomic mass unit, symbol u, is equal to 1.6605 ´ 10–27 kg.
- 1 Show that the mass decreases in this reaction.
  Calculate Δm in atomic mass units and in kilograms.

$$\text{Mass of H plus Li} = 1.0073 \text{ u} + 7.0160 \text{ u} = 8.0233 \text{ u}$$

$$\text{Mass of two He} = 2 \times 4.0015 \text{ u} = 8.0030 \text{ u}$$

$$\text{Difference } \Delta m = 8.0030 \text{ u} - 8.0233 \text{ u} = -0.0203 \text{ u}$$

So we can find the mass difference in kg:

$$\Delta m = -0.0203 \text{u} \times 1.6605 \times 10^{-27} \text{ kg} = -3.3708 \times 10^{-29} \text{ kg}$$

Q5

- proton: 1.0073 u
- neutron: 1.0087 u
- deuteron: 2.0136 u.
- Calculate the difference in mass between a deuteron and one proton and one neutron.

The mass difference is:

$$2.0136\ \text{u} - (1.0073\ \text{u} + 1.0087\ \text{u}) = -0.0024\ \text{u}.$$

n kg the mass difference is:

$$-0.0024\ \text{u} \times (1.66 \times 10^{-27}\ \text{kg}) = -3.98 \times 10^{-30}\ \text{kg}$$

Q6

- Calculate the binding energy of the deuteron in J and in MeV.

$$\text{Binding energy} = -3.98 \times 10^{-30}\ \text{kg} \times (3 \times 10^{8}\ \text{m s}^{-1})^2$$

$$= -3.58 \times 10^{-13}\ \text{J}$$

$$= -\frac{3.58 \times 10^{-13}\ \text{J}}{1.6 \times 10^{-19}\ \text{J eV}^{-1}} = -2.2 \times 10^{6}\ \text{eV}$$

$$= -2.2\ \text{MeV}.$$

Q7

- Calculate the binding energy per nucleon of the deuteron.

The deuteron has two nucleons so the binding energy per nucleon is

–2.2 MeV / 2 = –1.1 MeV.

Q8

- Express the difference in mass as a percentage of the sum of the masses of the proton and neutron.

As a percentage the mass difference is equal to:

$$\frac{0.0024\ \text{u}}{1.0073\ \text{u} + 1.0087\ \text{u}} = 1.2 \times 10^{-3} \times 100 = 0.1\%\ \text{(approximat ely)}$$

Q9

A possible reaction for the nuclear fission of uranium-235 is:

$$^{235}_{92}U + {}^{1}_{0}n \rightarrow {}^{133}_{51}Sb + {}^{99}_{41}Nb + 4{}^{1}_{0}n.$$

The masses of the particles are

- U-235 = 235.0439 u
- Sb-133 = 132.9152 u
- Nb-99 = 98.9116 u
- neutron (n) = 1.0087 u.

Show that the energy change per atom of uranium is about 200 MeV and calculate $\Delta m/m$.

Q9 continued

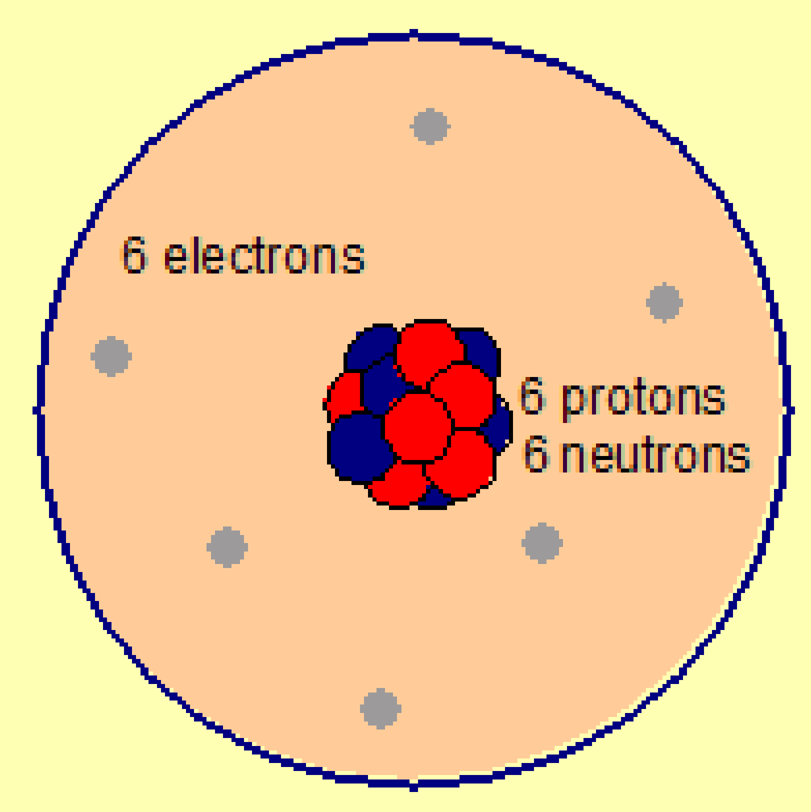
$$\text{Mass after } = 132.9152\,\text{u} + 98.9116\,\text{u} + (4\times 1.0087\,\text{u}) = 235.8616\,\text{u}$$

$$\text{Mass difference } = 236.0526\,\text{u} - 235.8616\,\text{u} = 0.191\,\text{u}.$$

$$\text{Change in rest energy } = \frac{0.191\,\text{u}\times(1.66\times 10^{-27}\ \text{kg})\times(3\times 10^{8}\ \text{m s}^{-1})^{2}}{1.6\times 10^{-19}\ \text{J eV}^{-1}} = 1.78\times 10^{8}\ \text{eV} = 178\ \text{MeV}.$$

The ratio is given by:

$$\Delta m / m = 0.191\,\text{u} / 236\,\text{u} = 8.1\times 10^{-4} \sim 0.1\%.$$



Mass of carbon-12 atom

1 atomic mass unit u

= 1/12 of mass of C-12 atom

1 u = 1.66056×10^{-27} kg

mass of C-12 atom = 12.0 u

Mass of 6 electrons

mass of electron

= 9.1095×10^{-31} kg

= 0.000 549 u

mass of 6 electrons = 0.0033 u

Difference in mass

= mass of carbon-12 nucleus – mass of protons and neutrons

= (11.9967 – 12.0956) u

= – 0.0989 u

Binding energy

in mass units:

= – 0.0989 u

= – 1.643 × 10^{-28} kg

$E_{rest} = mc^2$

in energy units:

= – 1.477 × 10^{-11} J

= – 92.16 MeV

Binding energy per nucleon

–92.16 MeV for 12 nucleons

= – 7.7 MeV per nucleon

The kettle contains 1 litre of water. The data you need are listed below.

1 atomic mass unit (u) = 931 MeV

1 eV = 1.6×10^{-19} J

Particle	Mass / u
$^{1}_{1}H$	1.007 825
$^{2}_{1}H$	2.014 102
$^{3}_{2}He$	3.016 030
$^{1}_{0}n$	1.008 665

Q10

Two deuterium nuclei ${}^{2}_{1}H$ can fuse to give one nucleus of helium ${}^{3}_{2}He$ with the ejection of one other particle. Write down the balanced equation that represents this reaction.

$${}^{2}_{1}H + {}^{2}_{1}H \rightarrow {}^{3}_{2}He + {}^{1}_{0}n$$

Q11

- Calculate the mass change that occurs in this reaction

$$\Delta m = (3.016\,030\ \text{u} + 1.008\,665\ \text{u}) - 2 \times 2.014\,102\ \text{u} = -0.0035\ \text{u}$$

Q12

- Convert this energy into joules.

$$0.003509\ \text{u} \times 931 \times 10^{6}\ \text{eV u}^{-1} \times 1.6 \times 10^{-19}\ \text{J eV}^{-1} = 5.23 \times 10^{-13}\ \text{J}$$

Q13

- *Use these values to solve the following problems:* mass of hydrogen 1.007825 u,
- mass of neutron 1.008665 u, 1 u 931.49 MeV.

The carbon isotope $^{12}_{6}C$ has a mass of 12.0000 u.

Mass defect = (isotope mass) − (mass of protons and electrons) − (mass of neutrons)

$$= 12.000000\ u - (6)(1.007825\ u) - (6)(1.008665\ u)$$
$$= -0.098940\ u$$

Q13 continued

Calculate its binding energy in MeV.

Binding energy = (mass defect)(binding energy of 1 u)

$$= (-0.098940\ \text{u})(931.49\ \text{MeV/u})$$

$$= -92.161\ \text{MeV}$$

Q14