

Half Life Equations

Term 3 Week 2

The decay constant, λ

The decay constant λ of a radio-nuclide is the probability that an individual nucleus will decay within a unit of time.

Since λ is the chance per unit time, its unit is s^{-1} , h^{-1} , day^{-1} , etc.

If the value of λ for a particular nuclide is 0.2 s^{-1} , then in one second, each nucleus has a 0.2 or $\frac{1}{5}$ chance of decay.

In a sample of 1000 nuclei, 200 will have decayed after 1 second.

Activity

The activity A of a source is the number of its nuclei that decay in unit time.

Since each decay produces ionising radiation, the activity of a source also tells you the number of ionising particles that it emits in unit time.

1 Bq is an activity of 1 decay per second. $1 \text{ Bq} = 1 \text{ s}^{-1}$.

The activity of a sample must depend on the decay constant λ of the nuclide.

$$\text{Activity, } A \text{ (s}^{-1}\text{)} = \text{decay constant, } \lambda \text{ (s}^{-1}\text{)} \times \text{number of undecayed nuclei, } N$$

$$\mathbf{A = \lambda N}$$

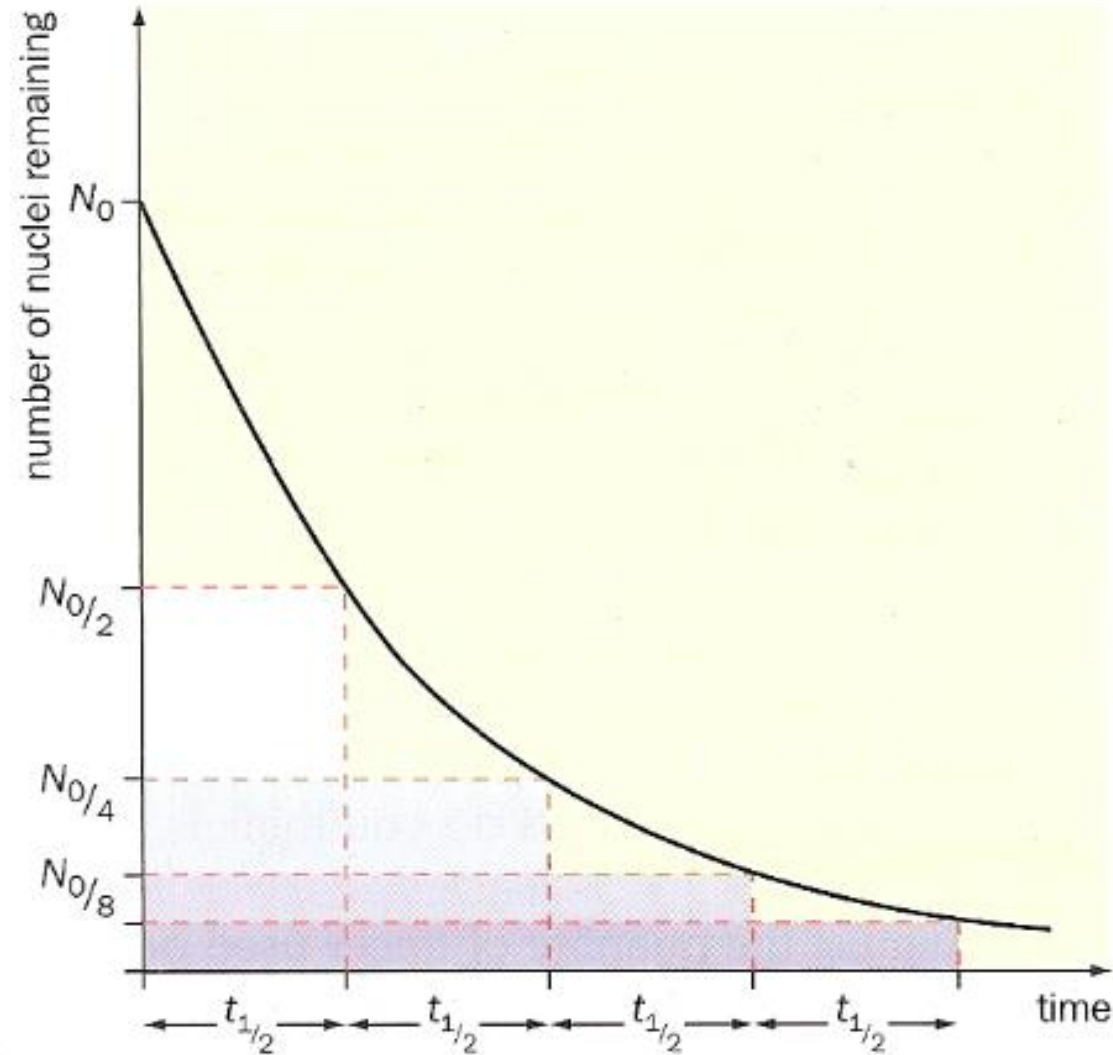
Example 1

A sample of a radio-nuclide initially contains 200 000 nuclei. Its decay constant is 0.30 s^{-1} . What is the initial activity?

$$A = \lambda N$$

$$A = 0.30 \text{ s}^{-1} \times 200\,000 = \underline{60\,000 \text{ s}^{-1}}$$

Exponential decay



This graph is an **exponential decay curve** because the number of nuclei always falls by the same fraction in the same time.

The equation for this curve is:

$$N = N_0 e^{-\lambda t}$$

N = the number of nuclei at time t
 N_0 = the original number of nuclei
 λ = the decay constant

$$A = A_0 e^{-\lambda t}$$

since the activity A is proportional to the number of nuclei present ($A = \lambda N$)

Half Life

$$\frac{1}{2^n} = \frac{A}{A_0}$$

A = number of nuclei remaining

A_0 = original number of nuclei

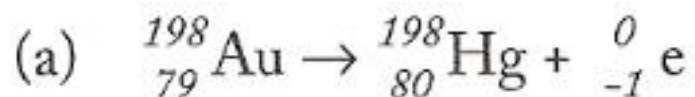
n = number of half lives

Example 2

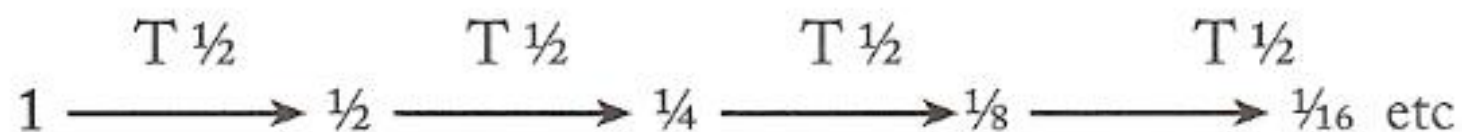
The radioactive isotope gold-198 is a β^- -emitter and is often used as a radioisotopic tracer. It can be used, for instance, by health engineers to trace the flow of sewage. It is found that its activity drops to $\frac{1}{8}$ of its original value in just over 8 days.

- (a) Write the β^- -decay equation for this isotope (${}_{79}^{198}\text{Au}$). You will need to refer to a periodic table.
- (b) Calculate its approximate half-life.
- (c) Estimate the fraction of gold-198 remaining after 2 weeks.
- (d) Why do you think gold-198 may be chosen as a tracer to study the flow of sewage?

Example 2 continued



(b) Activity drops off as follows:



Hence, to drop to $\frac{1}{8}$ of its original value will take 3 half-lives.

Hence, half-life of gold-198 $\approx \frac{8 \text{ days}}{3} \approx 2.7$ days

(c) After 2 weeks, 6 half-lives would have occurred for gold 198.

Hence fraction remaining = $(\frac{1}{2}) \times (\frac{1}{2}) \times (\frac{1}{2}) \times (\frac{1}{2}) \times (\frac{1}{2}) \times (\frac{1}{2}) = \frac{1}{64}$

(d) Since gold-198 is a β -emitter its presence can be detected through liquids or thin metal. The relatively short half-life of gold-198 means that:

(i) it is a fairly active isotope and more easily detectable;

(ii) it does not remain in the system for too long since its activity is effectively zero after a few weeks.

Example 3

A sample of processed waste from a nuclear reactor contains 1.0×10^{24} plutonium-239 atoms. The half-life of plutonium-239 is 2.41×10^4 years.

[a] How many plutonium-239 atoms will decay in the next 2.41×10^4 years?

[b] Estimate the activity of the original sample.

[c] Write an equation for the alpha decay of Pu-239 to uranium-235.

A half will decay in one half life so atoms decaying = $\frac{1}{2} \times 1 \times 10^{24} = 5.0 \times 10^{23}$

$$\text{Activity, } A = \lambda \times N \quad \lambda = \frac{\ln(2)}{t_{1/2}} = \frac{0.693}{2.41 \times 10^4 \text{ y}} = 2.88 \times 10^{-5} \text{ y}^{-1}$$



Example 4

The half life of a radioactive cobalt-60 source radiographers use for medical irradiation is 5.3 years.

[a] If the activity of a new source is 800 GBq, what is its activity after 26 years?

[b] If it is not safe to dispose of the source till the radioactivity is less than one thousandth of its original activity (i.e. 800 MBq), estimate to the nearest decade when this will be.

$$\frac{A}{A_0} = 0.5^n$$

where n is the number of half lives which have elapsed = $\left(\frac{t}{t_{1/2}}\right)$ and A_0 is the original activity

so the new activity, $A = 0.5^{\left(\frac{26 \text{ y}}{5.3 \text{ y}}\right)} \times 800 \text{ GBq} = 26.8 \text{ GBq}$

Example 5

Magnesium-24 (${}_{12}^{24}\text{Mg}$) is a stable nuclide. It can be transformed into sodium-24 (${}_{11}^{24}\text{Na}$) by neutron bombardment. Sodium-24 is radioactive, being a beta and gamma emitter with a half-life of 15 hours. It is used as a tracer in blood circulation studies. It decays back to magnesium-24.

[b] If 1.0 g of sodium-24 is injected into the blood of a patient, approximately how long will it take for 90% of this amount to decay?

if 90% has decayed, then 10% remains, so $\frac{A}{A_0} = 0.5^n = 0.10$

$$\text{then } n = \frac{\ln(0.10)}{\ln(0.5)} = 3.32$$

$$\text{Since } n = \frac{t}{t_{1/2}}$$

$$\text{then } t = n \times t_{1/2} = 3.32 \times 15 \text{ h} = 49.8 \text{ hours}$$

Example 6

A large amount of strontium-90 was released during the Chernobyl nuclear reactor accident in 1986. Strontium-90 enters the body through the food chain and accumulates in the bones. How long will it take for 95% of the Sr-90 released during the accident to decay? The half-life of Sr-90 is 28.9 years.

if 95% has decayed, then 5% remains, so $\frac{A}{A_0} = 0.5^n = 0.05$

$$\text{then } n = \frac{\ln(0.05)}{\ln(0.5)} = 4.32$$

$$\text{Since } n = \frac{t}{t_{1/2}}$$

$$\text{then } t = n \times t_{1/2} = 4.32 \times 28.9 \text{ y} = 125 \text{ years}$$

- $\frac{A}{A_0} = 0.5^n$

A large amount of strontium-90 was released during the Chernobyl nuclear reactor accident in 1986. Strontium-90 enters the body through the food chain and accumulates in the bones. How long will it take for 95% of the Sr-90 released during the accident to decay? The half-life of Sr-90 is 28.9 years.

- $\frac{0.1}{1.0} = 0.5^n$

[b] If 1.0 g of sodium-24 is injected into the blood of a patient, approximately how long will it take for 90% of this amount to decay?

if 90% has decayed, then 10% remains, so $\frac{A}{A_0} = 0.5^n = 0.10$

- $0.1 = 0.5^n$

- $\ln(0.1) = \ln(0.5^n)$

- $\ln(0.1) = n \ln(0.5)$

- $n = \frac{\ln(0.1)}{\ln(0.5)}$

then $n = \frac{\ln(0.10)}{\ln(0.5)} = 3.32$

Since $n = \frac{t}{t_{1/2}}$

then $t = n \times t_{1/2} = 3.32 \times 15 \text{ h} = 49.8 \text{ hours}$