2014 Term 3 Week 2

Mass Defect

Notes

- Distinguish between the **atomic mass** and the **nuclear mass**
- The atomic mass is the mass of an atom complete with its electrons
- The nuclear mass is the mass of the nucleus alone

- What is the nuclear mass of helium 3 (^3He) of which the atomic mass is 3.016030 u?
- There are 2 electrons as the proton number for helium is 2.
- We need to take away the mass of two electrons
- Nuclear mass = 3.016030 u – (2 × 0.000549 u) = **3.014932 u**

Mass Defect

- All atoms are lighter than the sum of the masses of the protons, electrons, and neutrons.
- This is the **mass defect**, which is the difference between the total mass of the nucleons and the measured mass of the nucleus itself.

Binding Energy

- To extract a proton or a neutron from the nucleus, we have to pull pretty hard.
- Then we find that it will regain its missing mass.
- We can use the idea of **binding energy** to explain this.
- The binding energy is defined as the energy released when a nucleus is assembled from its constituent nucleons.
- It is equal to the energy needed to tear the nucleus apart into its nucleons.

- Convert the mass from atomic mass units to kilograms
- What is the binding energy of the helium atom whose mass defect is 0.030377 u?
- $m = 0.030377 \text{ u} \times 1.661 \times 10^{-27} \text{ kg} = 5.046 \times 10^{-27} \text{ kg}$
- $E = mc^2 = 5.046 \times 10^{-27} \text{ kg} \times (3 \times 10^8)^2 = 4.541 \times 10^{-12} \text{ J}$

- Joules are not convenient units to use at the nuclear level, so we convert to electron volts (eV) by dividing by 1.6×10^{-19} J/eV.

- A useful conversion factor between mass and energy is that 1 u = 931.3 MeV

Particle	Mass (u)	Number	Total (u)
Proton	1.007276	3	3.021828
Neutron	1.008665	4	4.03466

What is the mass defect in atomic mass units (u) and in kilograms for the lithium nucleus which has 7 nucleons, and a proton number of 3?

What is the binding energy in J and eV?

What is the binding energy per nucleon in eV? The nuclear mass = 7.014353 u.

Li has a nucleon number of 7 and a proton number of 3, which means there are 3 protons and 4 neutrons

Now look up the masses for the proton and neutron from the data.

Add them together to get 7.056488 u

- Now take away the nuclear mass from the number above to get the mass deficit.
- 7.056488 u - 7.014353 u = 0.042135 u
- Now convert to kilograms: 1 u = 1.661×10^{-27} kg
- 0.042135 u × 1.661×10^{-27} kg = $6.9986235 \times 10^{-29}$ kg
- Now use $E = mc^2$ to work out the binding energy:
- E = $6.9986235 \times 10^{-29}$ kg × $(3 \times 10^{8}$ m/s$)^2$ = 6.3×10^{-12} J
- In electron volts, this is 6.3×10^{-12} J ÷ 1.6×10^{-19} eV/J = 3.9×10^{7} eV = 39 MeV.
- There are 7 nucleons so the binding energy per nucleon = 3.9×10^{7} eV ÷ 7 = 5.6×10^{6} eV

- What is the mass defect in atomic mass units (u) and in kilograms for the copper nucleus which has 63 nucleons, and a proton number of 29?
- What is the binding energy in J and eV?
- What is the binding energy per nucleon in eV?

The nuclear mass = 62.91367 u

- Number of protons = 29; number of neutrons = 63 – 29 = 34.
- Mass of protons = 29 $\times$ 1.007276 = 29.211004 u
- Mass of neutrons = 34 $\times$ 1.008665 = 34.29461 u
- Total mass = 29.211004 u + 34.29461 u = 63.505614 u
- Mass defect = 63.505614 u – 62.91367 u = 0.591944 u
- Mass defect in kg = 0.591944 $\times$ 1.661 $\times 10^{-27}$ = 9.83218 $\times 10^{-28}$ kg
- Binding energy = mc^2 = 9.83218 $\times 10^{-28}$ kg $\times$ (3 $\times 10^{8}$ m/s)^2 = 8.85 $\times 10^{-11}$ J
- Binding energy in eV = 8.85 $\times 10^{-11}$ J $\div$ 1.6 $\times 10^{-19}$ J/eV = 5.53 $\times 10^{8}$ eV
- Binding energy per nucleon = 5.53 $\times 10^{8}$ eV $\div$ 63 = 8.78 $\times 10^{6}$ eV

$$^{228}_{90}Th \rightarrow {}^{224}_{88}Ra + {}^{4}_{2}\alpha$$

- Mass of the thorium nucleus = 227.97929 u
- Mass of the radium nucleus = 223.97189 u
- Mass of the alpha particle (helium nucleus) = 4.00151 u
- Mass on the left hand side = 227.97929 u
- Mass on the right hand side = 223.97189 u + 4.00151 u = 227. 97340 u
- The right hand side has a mass defect = 227.97929 u - 227. 97340 u = 0.00589 u

- The mass defect can be written in kilograms and the energy can be expressed in joules
- The energy equivalence of 1 u = 931.5 MeV.
- So the energy given out by this decay is:
- $E = 931.5 \times 0.00589 = 5.49$ MeV.

$$^{29}_{13}Al \rightarrow {}^{29}_{14}Si + {}^{0}_{-1}e^{-} + {}^{0}_{0}\bar{\nu}_{e}$$

- Mass of aluminium nucleus = 28.97330 u
- Mass of silicon nucleus = 28.96880 u
- Mass of beta particle (electron) = 0.00549 u
- Mass of electron antineutrino = 0
- What is energy given out by the above decay in MeV?

- Mass on right hand side = 28.96880 u + 0.000549 u + 0 = 28.969349 u
- Mass defect = 28.97330 u - 28.969349 u = 0.0039510 u
- Energy = 931.5 $\times$ 0.0039510 u = 3.68 MeV

$$^{2}_{1}H + ^{3}_{1}H \rightarrow ^{4}_{2}He + ^{1}_{0}n + energy$$

- Mass of deuterium nucleus = 3.3425×10^{-27} kg
- Mass of tritium nucleus = 6.6425×10^{-27} kg
- Mass of helium nucleus = 6.6465×10^{-27} kg
- Mass of a neutron = 1.675×10^{-27} kg
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- What is the energy in J and eV released in this reaction above?

- Mass on the left hand side = 3.3425×10^{-27} kg + 6.6425×10^{-27} kg = 9.985×10^{-27} kg
- Mass on right hand side = 6.6465×10^{-27} kg + 1.675×10^{-27} kg = 8.3215×10^{-27} kg
- Mass deficit = 9.985×10^{-27} kg - 8.3215×10^{-27} kg = 1.6635×10^{-27} kg
- Energy = 1.6635×10^{-27} kg $\times (3.0 \times 10^{8}$ m/s$)^2$ = 1.50×10^{-11} J
- Energy in eV = 1.50×10^{-11} J $\div 1.6 \times 10^{-19}$ = 9.4×10^{8} eV = 940 MeV