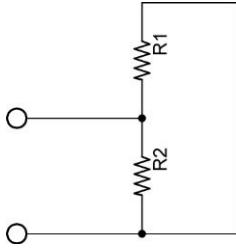
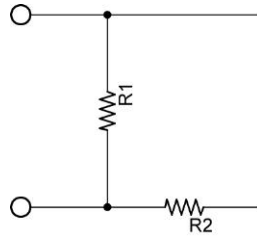


- **Parallel Circuits:**

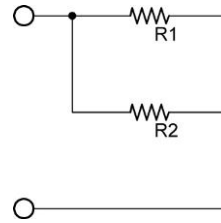
- A parallel circuit is one that has more than one pathway for the electrons to flow. Unlike series circuit, when you remove a resistor in a parallel circuit, electrons continue to flow.
- Identifying parallel circuits: Each example below is a circuit with two parallel paths; all of the circuit configurations are electrically equivalent to each other.



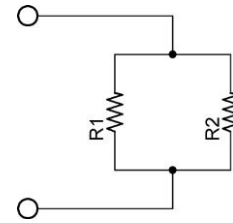
**Circuit 1**



**Circuit 2**



**Circuit 3**



**Circuit 4**

- Everyday examples of parallel circuits:
  - Lights in a home
  - Electrical car functions, such as the radio, horn, starter, lights, etc.
- **Voltage in a Parallel Circuit:**
  - If components are connected in parallel to the source, the voltage drop across each component is the same as the source voltage.
  - Mathematically:

$$V_T = V_1 = V_2 = V_3 = \dots V_N$$

Where:

$V_T$  = Total voltage applied to the series circuit

$V_1$  = Voltage drop across  $R_1$

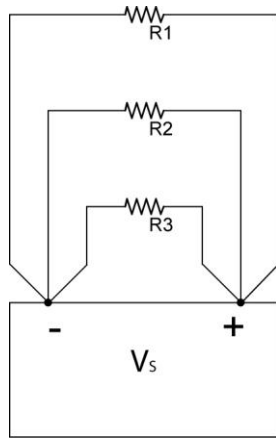
$V_2$  = Voltage drop across  $R_2$

$V_3$  = Voltage drop across  $R_3$

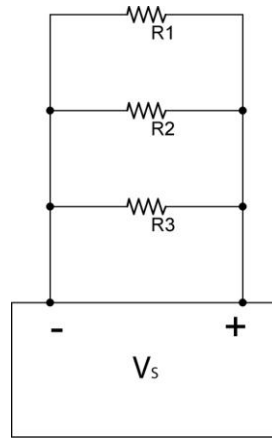
$V_N$  = Voltage drop across  $R_N$

$N$  = The number of resistors in the series

- The voltage drop across each component (resistors in this case) is the same. In the two circuits below, the connections in Figure 1 and Figure 2 are electrically equivalent.



**Figure 1**



**Figure 2**

○ **Current in a Parallel Circuit:**

- Kirchhoff's Current Law: The sum of the currents into a junction is equal to the sum of the currents out of that junction.
- Mathematically:

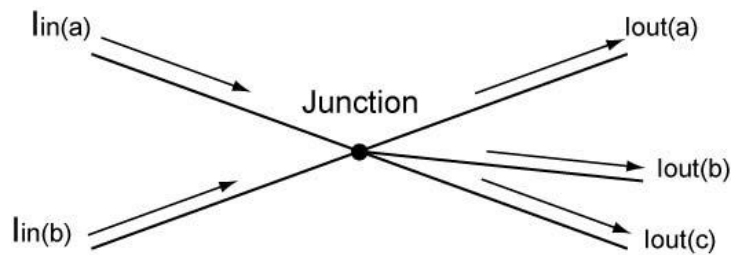
$$I_{\text{TOTAL in}} = I_{\text{TOTAL out}}$$

Where:

$I_{\text{TOTAL in}}$  = the sum of currents into a junction

$I_{\text{TOTAL out}}$  = the sum of currents out of a junction

For Example:



**Figure 3**

$$I_{\text{in}(a)} + I_{\text{in}(b)} = I_{\text{out}(a)} + I_{\text{out}(b)} + I_{\text{out}(c)}$$

Another way of expressing Kirchoff's Current Law (see Figure 4):

$$I_T = I_1 + I_2 + I_3 + \dots + I_N$$

Where:

$I_T$  = Total current into a parallel resistor circuit

$I_1$  = Current through  $R_1$

$I_2$  = Current through  $R_2$

$I_3$  = Current through  $R_3$

$I_N$  = Current through  $R_N$

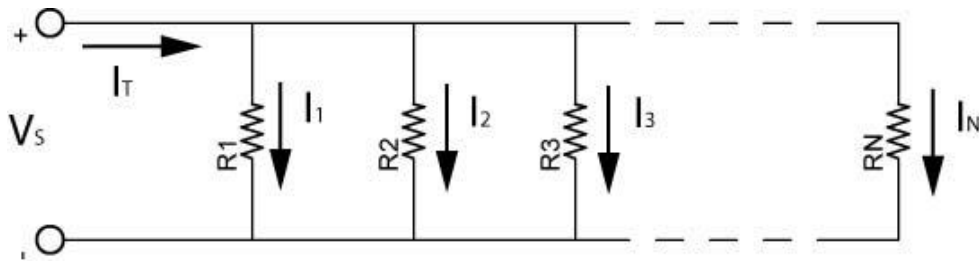
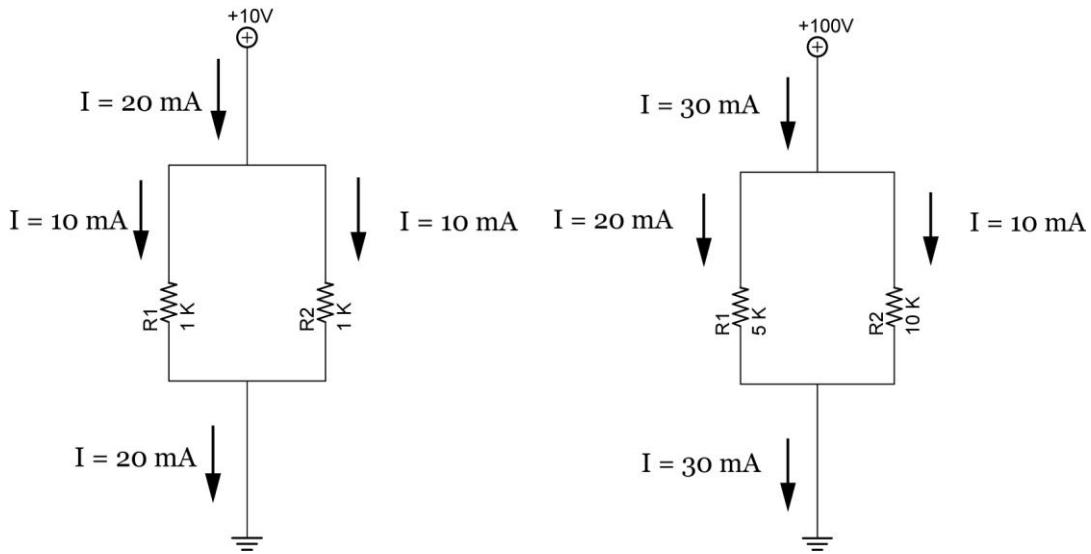


Figure 4

- Parallel circuits act as current dividers. See the two examples below.



Example 1

Example 2

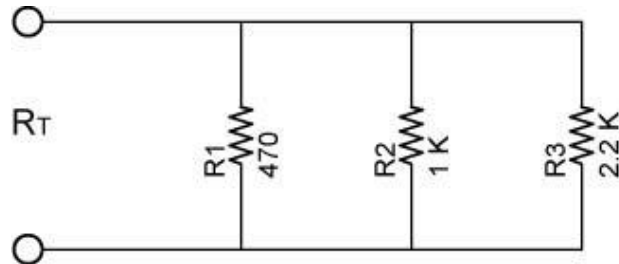
- Resistance in Parallel Circuits:
  - When resistors are connected in parallel circuits, the total resistance is always less than the value of the smallest resistor.
  - Reciprocal Rule:

- The reciprocal of a number is equal to 1 divided by that number, e.g., the reciprocal of 4 is  $\frac{1}{4}$ , and the reciprocal of 87 is  $\frac{1}{87}$ .
- The total resistance of a parallel circuit is:

$$1/R_T = 1/R_1 + 1/R_2 + 1/R_3 + \dots + 1/R_N$$

Where  $R_T$  is the total resistance and  
 $N$  is the total number of resistors in parallel.

- For example, find the total resistance in Circuit 5:



**Circuit 5**

$$1/R_T = 1/R_1 + 1/R_2 + 1/R_3$$

$$1/R_T = 1/470 + 1/1000 + 1/2200$$

$$1/R_T = 0.0021 + 0.001 + 0.0004$$

$$1/R_T = 0.0035$$

$$R_T = 1/0.0035$$

$$R_T = 286 \Omega$$

- Special Case 1: Two resistor parallel circuit: If  $1/R_T = 1/R_1 + 1/R_2$ , then

$$R_T = R_1 R_2 / (R_1 + R_2)$$

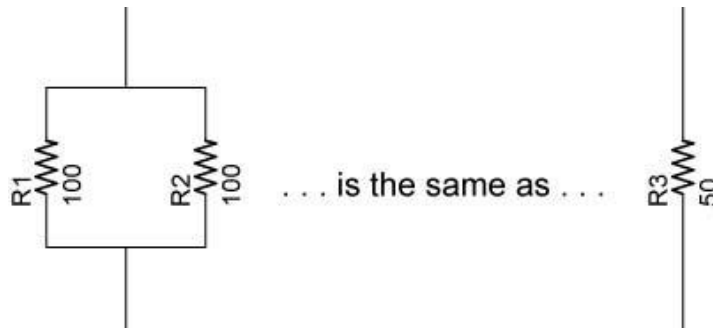
- Special Case 2: Resistors of equal value:

$$R_T = R/N$$

Where:

R = the value of each resistor (all being the same)  
 N = the number of resistors

For Example:



$$R_T = R/N$$

$$R_T = 100/2$$

$$R_T = 50 \Omega$$

How many 1 K resistors in parallel would you need to create a total resistance of 1 ohm?

- In a parallel circuit, a resistor that is much smaller than the other resistors dominates.
- Applet: <http://www.lon-capa.org/~mmp/kap20/RR506a.htm>
- **Power in a Parallel Circuit:**
  - The total power is equal to the sum of all the power of each resistor in the parallel circuit.

$$P_T = P_1 + P_2 + P_3 + \dots + P_N$$

Where  $P_T$  is the total power consumed in the circuit and  
 N is the total number of resistors in parallel.

- Power is also equal to the source voltage times the total current.

$$P_T = V_T \times I_T$$

Where  $P_T$  is the total power consumed in the circuit,  
 $V_T$  is the source voltage, and  
 $I_T$  is the total current

- Parallel Circuits:
  - Solving for Resistance, Voltage, and Current in a Parallel Resistor Circuits:
    - Four equations are used to solve parallel resistor circuits. They are:

$$1/R_T = 1/R_1 + 1/R_2 + 1/R_3 + \dots + 1/R_N$$

$$V_T = V_1 = V_2 = V_3 = \dots = V_N$$

$$I_T = I_1 + I_2 + I_3 + \dots + I_N$$

$$V = I \times R$$

$V = I \times R$  can be applied to the total circuit ( $V_T = I_T \times R_T$ ) and to individual resistors ( $V_1 = I_1 \times R_1$ ).

- A table will be used to help solve our circuits. To begin, a table as shown in Table 1 corresponds to the circuit in Figure 5:

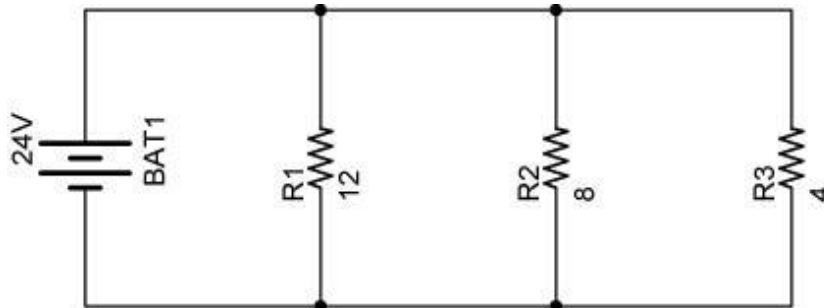


Figure 5

	Resistance	Voltage	Current
R1	R1 = 12 Ω		
R2	R2 = 8 Ω		
R3	R3 = 4 Ω		
Total		V <sub>T</sub> = 24 V	

Table 1

- Table 2 lists all of the unknowns that will be solved.

	Resistance	Voltage	Current
R1	R1 = 12 Ω	V1	I1
R2	R2 = 8 Ω	V2	I2
R3	R3 = 4 Ω	V3	I3
Total	R <sub>T</sub>	V <sub>T</sub> = 24 V	I <sub>T</sub>

Table 2

- Step 1: Find V<sub>1</sub>, V<sub>2</sub>, and V<sub>3</sub>.

$$V_T = V_1 = V_2 = V_3 = 24 \text{ V}$$

See Table 3:

	<b>Resistance</b>	<b>Voltage</b>	<b>Current</b>
R1	R1 = 12 $\Omega$	V1 = 24 V	I <sub>1</sub>
R2	R2 = 8 $\Omega$	V2 = 24 V	I <sub>2</sub>
R3	R3 = 4 $\Omega$	V3 = 24 V	I <sub>3</sub>
Total	R <sub>T</sub>	V <sub>T</sub> = 24 V	I <sub>T</sub>

**Table 3**

- Step 2: Find I<sub>1</sub>, I<sub>2</sub>, and I<sub>3</sub>.

$$V_1 = I_1 \times R_1, \text{ therefore,}$$

$$I_1 = V_1 / R_1$$

$$I_1 = 24 \text{ V} / 12 \Omega$$

$$I_1 = 2 \text{ A}$$

$$I_2 = V_2 / R_2$$

$$I_2 = 24 \text{ V} / 8 \Omega$$

$$I_2 = 3 \text{ A}$$

$$I_3 = V_3 / R_3$$

$$I_3 = 24 \text{ V} / 4 \Omega$$

$$I_3 = 6 \text{ A}$$

See Table 4:

	<b>Resistance</b>	<b>Voltage</b>	<b>Current</b>
R1	R1 = 12 $\Omega$	V1 = 24 V	I1 = 2 A
R2	R2 = 8 $\Omega$	V2 = 24 V	I2 = 3 A
R3	R3 = 4 $\Omega$	V3 = 24 V	I3 = 6 A
Total	R <sub>T</sub>	V <sub>T</sub> = 24 V	I <sub>T</sub>

**Table 4**

- Step 3: Find  $I_T$ .

$$I_T = I_1 + I_2 + I_3$$

$$I_T = 2 \text{ A} + 3 \text{ A} + 6 \text{ A}$$

$$I_T = 11 \text{ A}$$

See Table 5:

	<b>Resistance</b>	<b>Voltage</b>	<b>Current</b>
R1	R1 = 12 $\Omega$	V1 = 24 V	I1 = 2 A
R2	R2 = 8 $\Omega$	V2 = 24 V	I2 = 3 A
R3	R3 = 4 $\Omega$	V3 = 24 V	I3 = 6 A
Total	$R_T$	$V_T = 24 \text{ V}$	$I_T = 11 \text{ A}$

**Table 5**

- Step 5: Find  $R_T$ .

$$1/R_T = 1/R_1 + 1/R_2 + 1/R_3$$

$$1/R_T = 1/12 + 1/8 + 1/4$$

$$1/R_T = 0.083 + 0.125 + 0.25$$

$$1/R_T = 0.458$$

$$R_T = 1/0.458$$

$$R_T = 2.18 \Omega$$

Or an alternate way:

$$V_T = I_T \times R_T, \text{ therefore,}$$

$$R_T = V_T / I_T$$

$$R_T = 24 \text{ V} / 11 \text{ A}$$

$$R_T = 2.18 \Omega$$

See Table 6:

	<b>Resistance</b>	<b>Voltage</b>	<b>Current</b>
R1	R1 = 12 $\Omega$	V1 = 24 V	I1 = 2 A
R2	R2 = 8 $\Omega$	V2 = 24 V	I2 = 3 A
R3	R3 = 4 $\Omega$	V3 = 24 V	I3 = 6 A
Total	$R_T = 2.18 \Omega$	$V_T = 24 \text{ V}$	$I_T = 11 \text{ A}$

**Table 6**

- Since all of the resistances, voltages, and currents are solved in the present problem, the power can now be calculated.

- Solving for Power in a Parallel Resistor Circuits:
  - Two equations are used to solve for power in a parallel resistor circuit. They are:

$$P_T = P_1 + P_2 + P_3 + \dots + P_N$$

$$P = V \times I$$

$P = V \times I$  can be applied to the total circuit ( $P_T = V_T \times I_T$ ) and to individual resistors ( $P_1 = V_1 \times I_1$ ).

- A column for power will be added to the table already used to solve our circuit. See Table 7.

	Resistance	Voltage	Current	Power
R1	R1 = 12 $\Omega$	V1 = 24 V	I1 = 2 A	P1
R2	R2 = 8 $\Omega$	V2 = 24 V	I2 = 3 A	P2
R3	R3 = 4 $\Omega$	V3 = 24 V	I3 = 6 A	P3
Total	R <sub>T</sub> = 2.18 $\Omega$	V <sub>T</sub> = 24 V	I <sub>T</sub> = 11 A	P <sub>T</sub>

**Table 7**

- Step 5: Solve for  $P_1$ ,  $P_2$ ,  $P_3$ , and  $P_T$ .

$$P_1 = V_1 \times I_1$$

$$P_1 = 24 \text{ V} \times 2 \text{ A}$$

$$P_1 = 48 \text{ W}$$

$$P_2 = V_2 \times I_2$$

$$P_2 = 24 \text{ V} \times 3 \text{ A}$$

$$P_2 = 72 \text{ W}$$

$$P_3 = V_3 \times I_3$$

$$P_3 = 24 \text{ V} \times 6 \text{ A}$$

$$P_3 = 144 \text{ W}$$

$$P_T = P_1 + P_2 + P_3$$

$$P_T = 48 \text{ W} + 72 \text{ W} + 144 \text{ W}$$

$$P_T = 264 \text{ W}$$

See Table 8:

	Resistance	Voltage	Current	Power
R1	R1 = 12 $\Omega$	V1 = 24 V	I1 = 2 A	P1 = 48 W
R2	R2 = 8 $\Omega$	V2 = 24 V	I2 = 3 A	P2 = 72 W
R3	R3 = 4 $\Omega$	V3 = 24 V	I3 = 6 A	P3 = 144 W
Total	R <sub>T</sub> = 2.18 $\Omega$	V <sub>T</sub> = 24 V	I <sub>T</sub> = 11 A	P <sub>T</sub> = 264 W

**Table 8**